Exercise 41

For the following exercises, solve the equations over the complex numbers.

$$5x^2 + 6x + 2 = 0$$

Solution

Factor the coefficient of x^2.

$$5\left(x^2 + \frac{6}{5}x + \frac{2}{5}\right) = 0$$

The two terms with x, x^2 and $(6/5)x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = \frac{6}{5}$, which means $B = \frac{3}{5}$ and $B^2 = \frac{9}{25}$. Add and subtract $\frac{9}{25}$ within the parentheses on the left side and apply the identity.

$$5\left[\left(x^2 + \frac{6}{5}x + \frac{9}{25}\right) + \frac{2}{5} - \frac{9}{25}\right] = 0$$

$$5\left[\left(x + \left(\frac{3}{5}\right)\right)^2 + \frac{1}{25}\right] = 0$$

$$5\left(x + \frac{3}{5}\right)^2 + \frac{1}{5} = 0$$

Now that x appears in only one place, it can be solved for. Subtract $1/5$ from both sides.

$$5\left(x + \frac{3}{5}\right)^2 = -\frac{1}{5}$$

Divide both sides by 5.

$$\left(x + \frac{3}{5}\right)^2 = -\frac{1}{25}$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{\left(x + \frac{3}{5}\right)^2} &= \sqrt{-\frac{1}{25}} \\ &= \sqrt{\frac{1}{25}(-1)} \\ &= \sqrt{\frac{1}{25}}\sqrt{-1} \\ &= \frac{1}{5}i\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x + \frac{3}{5}$.

$$\left|x + \frac{3}{5}\right| = \frac{1}{5}i$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x + \frac{3}{5} = \pm\frac{1}{5}i$$

Subtract $\frac{3}{5}$ from both sides.

$$x = -\frac{3}{5} \pm \frac{1}{5}i$$

Therefore,

$$x = \left\{-\frac{3}{5} - \frac{1}{5}i, -\frac{3}{5} + \frac{1}{5}i\right\}.$$